Exam Kaleidoscope Modern Physics

31 october 2014, 9:00-12:00, A. Jacobshal

- Put your name and student number on each sheet.
- Answer all questions short and to the point, but complete; write legible.
- Use of a calculator is not allowed.
- $hc = 1240$ eV·nm ; $\hbar c = 200$ eV·nm .
- Final grade = total number of points/3 + 1

1. Ordinary stars shine because of nuclear fusion, producing elements heavier than hydrogen. However, no elements heavier than iron are produced in burning stars. Explain why. **(2 points)**
   Tests understanding of the concepts "binding energy" and "fusion". Tests ability to read and interpret the binding energy vs. atomic number diagram. The binding energy per nucleon steadily rises with atomic number A from hydrogen to iron. From that point on, the binding energy per nucleon descreases again with increasing A. As long as the binding energy rises when two lighter nuclei fuse, the Q-value is positive, and the reaction will go by itself. Beyond iron the binding energy decreases, leading to a negative Q-value. Hence energy must be put in to keep the reaction going. And hence no nuclei heavier than iron are produced.

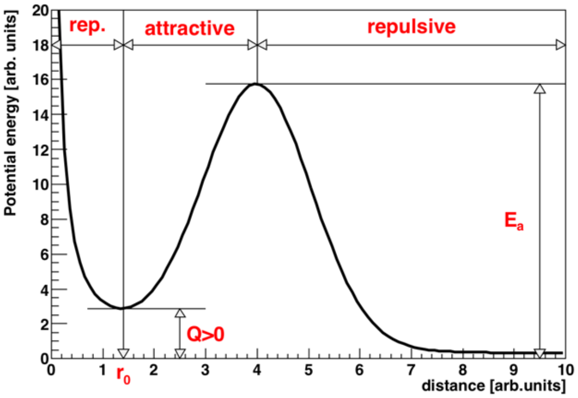
2. A free neutron has a mean life of 900 s. What is the (relative) uncertainty in its mass? **(2 points)**
   Tests the understanding of and ability to use the concept "uncertainty relation" and the ability to make use of the product $\hbar \cdot c$. Checks the reproduction of the prefix "femto". Use the uncertanty relation between energy and time, $\Delta E \Delta t \geq \hbar/2$. The lifetime can be used as a measure of the uncertainty Δt, so that $\Delta E \geq \hbar/2/\Delta t = \hbar c/(2\Delta t c) = 200\,\text{MeV} \cdot \text{fm}/(1800\,\text{s} \cdot 3 \times 10^8 \text{m/s}) \simeq 10^{-24}\,\text{MeV}$. The neutron mass corresponds to an energy of about $1\,\text{GeV} = 10^3\,\text{MeV}$, so the relative uncertainty is about 10^{-27}. Note that the answer is deliberately "weird" to avoid making the question too obvious.

3. Just like an electron, a negative muon may bind to a nucleus to form a so-called muonic atom. Carefully consider a Lithium atom with one of its electrons replaced by a muon. What is the lowest energy state the muon can be in? Explain your answer using the various quantum numbers involved. **(3 points)**
   Tests the understanding of the concept "exclusion principle", understanding of the distinguishability of electrons and muons (lepton number) and knowledge of atomic structure, in particular of the litium atom. The leptons are described by their lepton flavor. The set of quantum numbers describing the muon thus always differs from those of the electrons as $L_\mu \neq L_e$. The muonic states are, apart from the scaling factors, similar to the electronic states. Hence the state with the lowest energy is the $1S$-state, *i.e.* with $n = 1$ and thus $l = 0$.

4. Explain why a neutrino is so hard to detect. **(2 points)**
   Tests the reproduction of basic properties of the weak interaction and neutrinos. A neutrino is only subject to the weak interaction. The weak interaction has a very short range (approximately 0.001 fm (compared to 1 fm for the strong interaction and infinitely for the EM interaction) and hence the reaction cross section is extremely small. Detection requires a neutrino to somehow interact.

5. The ${}^{7}_{3}$Li nucleus has an excited state 0.48 MeV above the ground state. What wavelength gamma photon is emitted when the nucleus decays from the excited state to the ground state? **(2 points)**
   Tests the understanding of the concept "de Broglie wavelength", "ground state" and "excited state", the relation between the excitation energy and the energy of the emitted photon, and the ability to make use of the product $\hbar \cdot c$. The de Broglie relation says $\lambda = h/p$. For a photon $E = pc$, so $\lambda = pc/E = 1240\,\mathrm{MeV \cdot fm}/0.48\,\mathrm{MeV} = 2500\,\mathrm{fm} = 2.5\,\mathrm{pm}$.

6. Sketch the potential energy vs atomic distance diagram for a bond between two atoms, requiring an activation energy, with a respulsive core and which produces a *meta*-stable molecule. In your sketch, indicate the bond length r_0, the activation energy E_a, the Q-value, and the regions where the potential is repulsive or attractive. **(4 points)**
   Tests the understanding of the concept "potential", including several properties. A merger between figure 40-9 and 40-11. Activation energy: long distance behavior is repulsive, *i.e.* falls with increasing distance. Repulsive core: potential energy rises quickly for shorter ranges. Meta-stable: local minimum above zero around bond length r_0. The activation energy is the height above zero of the bump between the local minimum and infinity. The Q-value is the height of the dip at r_0 with respect to the potential at infinity (assumed to be zero; may have any value, but then all zero's above should be replaced by this value).



7. As ${}^{222}_{86}$Rn decays into ${}^{206}_{82}$Pb, how many alpha and beta particles are emitted? Does is matter which path in the decay series is chosen? Why (not)? **(2 points)**
   Tests understanding of the concepts "decay series", "α/β-decay" and reproduction of their properties, in particular conservation laws. In β decay A doesn't change. So the change in A from 222 to 206 can only be due to α emission. Since $\Delta A = A_{begin} - A_{end} = 16$, which would correspond to 4 α's. At the same time Z would change by 8 (for each α two protons leave the nucleus). Since $\Delta Z = 4$, an additional change of $\Delta Z = -4$ is necessary. This would correspond to the emission of 4 β^- particles. The precise path does not matter, as in the end only the conservation of charge and number of nucleons have to hold. There is a catch though: also β^+'s could be emitted. Then the condition holds that there are 4 more β^-'s emitted than β^+'s. In this case the path does matter.

8. Identify if atoms with the following electron-configurations exist. If so, give their name: (a) $1s^2 2s^2 2p^6 3s^2$; (b) $1s^2 2s^2 2p^6 3s^2 3p^6$; (c) $1s^2 2s^2 2p^8 3s^2 3p^6 4s^1$; (d) $1s^2 2s^2 2p^6 3s^2 3p^6 3d^6 4s^2$ **(3 points)**

   Tests the understanding of the concepts "atomic structure" and "shell structure", ability to reproduce atomic structure notation and their interpretation. Ability to relate atomic properties to the location in the periodic table. (a) Mg; (b) Ar; (c) doesn't exist; $2p^8$ has too many electrons. (d) Fe

9. Explain on the basis of the energy bands why the sodium chloride crystal is a good insulator. *Hint:* consider the shells of the Na^+ and Cl^- ions. **(2 points)**
   Tests the understanding of the concept "band structure", the relation between band gap and material properties, and the relation between atomic properties and solid state properties. Both Na^+ and Cl^- are in a noble gas configuration (a completely filled shell). Hence, in a solid the valence band is full. The conduction band originates from the next shell, and thus exhibits a large gap. This is the configuration of an insulator.

10. Fermi problem: How much ink is used to answer all the RUG exams during an exam week? **(5 points)**
    Several possible routes to solve this problem exist. Estimation of properties at hand have to be closer to reality. This includes for example the number of pages per exam, the number of students in a room, the number of exams per period or week, the amount of ink in a pen. Other quantities can be estimated less precisely. There are several things that you need to know: how many students are there at the RUG?; how many exams do they have in a week?; how much ink do they use per exam? Perhaps: how much ink is there in a pen? Step one: what is the number of students at the RUG. There are about 200,000 people in Groningen. They are not all associated to the university, but a fair share will be, assume 10%. Of those half will be students, the other half will be staff. So a total of 10,000 students would be realistic (28,000 according to RUG). Step two: the number of exams per student. From your own experience you know that you have two exams (Calculus I and Kaleidoscope; there is no exam for physics lab 1). Some students may have more exams (typically three), some may have less (none if you're doing research). So a typical number would be 1 per period and thus 1/2 per week. Taking 1 per week would no be too far off. So in total about 10,000 exams are done. Step three: An average pen lasts defintely more than 1 exam (if it doesn't get lost), but not for 100. If we use 10 exams, we're not too far off. So a total of 1,000 pens will be consumed (this is an acceptable answer, with number of pens as the unit). Each (ballpoint) pen has an ink volume of about 10 cm x 1 mm x 1 mm = $0.1\,\mathrm{cm}^3$ or 0.1 ml. So a total of 100 ml ink will be consumed (this is a better answer). Fountain pens use quite a bit more ink and contain about 1 ml. Answers around 1 l will be graded correct. Most importantly is the careful explanation of the various quantities needed, the use of units, and the avoidance of unnecessarily complicated calculations through the use of scientific notation.

Fundamental Constants

Quantity	Symbol	Approximate Value	Current Best Value[†]
Speed of light in vacuum	c	$3.00 \times 10^{8}\ \mathrm{m/s}$	$2.99792458 \times 10^{8}\ \mathrm{m/s}$
Gravitational constant	G	$6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$	$6.6728(67) \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$
Avogadro's number	N_A	$6.02 \times 10^{23}\ \mathrm{mol^{-1}}$	$6.02214179(30) \times 10^{23}\ \mathrm{mol^{-1}}$
Gas constant	R	$8.314\ \mathrm{J/mol \cdot K} = 1.99\ \mathrm{cal/mol \cdot K}$ $= 0.0821\ \mathrm{L \cdot atm/mol \cdot K}$	$8.314472(15)\ \mathrm{J/mol \cdot K}$
Boltzmann's constant	k	$1.38 \times 10^{-23}\ \mathrm{J/K}$	$1.3806504(24) \times 10^{-23}\ \mathrm{J/K}$
Charge on electron	e	$1.60 \times 10^{-19}\ \mathrm{C}$	$1.602176487(40) \times 10^{-19}\ \mathrm{C}$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}$	$5.670400(40) \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}$
Permittivity of free space	$\epsilon_0 = (1/c^2\mu_0)$	$8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$	$8.854187817\ldots \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$	$1.2566370614\ldots \times 10^{-6}\ \mathrm{T \cdot m/A}$
Planck's constant	h	$6.63 \times 10^{-34}\ \mathrm{J \cdot s}$	$6.62606896(33) \times 10^{-34}\ \mathrm{J \cdot s}$
Electron rest mass	m_e	$9.11 \times 10^{-31}\ \mathrm{kg} = 0.000549\ \mathrm{u}$ $= 0.511\ \mathrm{MeV}/c^2$	$9.10938215(45) \times 10^{-31}\ \mathrm{kg}$ $= 5.4857990943(23) \times 10^{-4}\ \mathrm{u}$
Proton rest mass	m_p	$1.6726 \times 10^{-27}\ \mathrm{kg} = 1.00728\ \mathrm{u}$ $= 938.27\ \mathrm{MeV}/c^2$	$1.672621637(83) \times 10^{-27}\ \mathrm{kg}$ $= 1.00727646677(10)\ \mathrm{u}$
Neutron rest mass	m_n	$1.6749 \times 10^{-27}\ \mathrm{kg} = 1.008665\ \mathrm{u}$ $= 939.57\ \mathrm{MeV}/c^2$	$1.674927211(84) \times 10^{-27}\ \mathrm{kg}$ $= 1.00866491597(43)\ \mathrm{u}$
Atomic mass unit (1 u)		$1.6605 \times 10^{-27}\ \mathrm{kg} = 931.49\ \mathrm{MeV}/c^2$	$1.660538782(83) \times 10^{-27}\ \mathrm{kg}$ $= 931.494028(23)\ \mathrm{MeV}/c^2$

[†] CODATA (3/07), Peter J. Mohr and Barry N. Taylor, National Institute of Standards and Technology. Numbers in parentheses indicate one-standard-deviation experimental uncertainties in final digits. Values without parentheses are exact (i.e., defined quantities).

Other Useful Data

Joule equivalent (1 cal)	4.186 J
Absolute zero (0 K)	−273.15°C
Acceleration due to gravity at Earth's surface (avg.)	$9.80\ \mathrm{m/s^2}\ (= g)$
Speed of sound in air (20°C)	343 m/s
Density of air (dry)	$1.29\ \mathrm{kg/m^3}$
Earth: Mass	$5.98 \times 10^{24}\ \mathrm{kg}$
Radius (mean)	$6.38 \times 10^{3}\ \mathrm{km}$
Moon: Mass	$7.35 \times 10^{22}\ \mathrm{kg}$
Radius (mean)	$1.74 \times 10^{3}\ \mathrm{km}$
Sun: Mass	$1.99 \times 10^{30}\ \mathrm{kg}$
Radius (mean)	$6.96 \times 10^{5}\ \mathrm{km}$
Earth–Sun distance (mean)	$149.6 \times 10^{6}\ \mathrm{km}$
Earth–Moon distance (mean)	$384 \times 10^{3}\ \mathrm{km}$

The Greek Alphabet

Alpha	A	α	Nu	N	ν
Beta	B	β	Xi	Ξ	ξ
Gamma	Γ	γ	Omicron	O	o
Delta	Δ	δ	Pi	Π	π
Epsilon	E	ϵ, ε	Rho	P	ρ
Zeta	Z	ζ	Sigma	Σ	σ
Eta	H	η	Tau	T	τ
Theta	Θ	θ	Upsilon	Υ	υ
Iota	I	ι	Phi	Φ	ϕ, φ
Kappa	K	κ	Chi	X	χ
Lambda	Λ	λ	Psi	Ψ	ψ
Mu	M	μ	Omega	Ω	ω

Values of Some Numbers

$\pi = 3.1415927$

$e = 2.7182818$

$\sqrt{2} = 1.4142136$

$\sqrt{3} = 1.7320508$

$\ln 2 = 0.6931472$

$\ln 10 = 2.3025851$

$\log_{10} e = 0.4342945$

$1\ \mathrm{rad} = 57.2957795°$

Mathematical Signs and Symbols

$\propto$	is proportional to	$\leq$	is less than or equal to
$=$	is equal to	$\geq$	is greater than or equal to
$\approx$	is approximately equal to	Σ	sum of
$\neq$	is not equal to	$\bar{x}$	average value of x
$>$	is greater than	Δx	change in x
$\gg$	is much greater than	$\Delta x \to 0$	Δx approaches zero
$<$	is less than	$n!$	$n(n-1)(n-2)\ldots(1)$
$\ll$	is much less than		

Properties of Water

Density (4°C)	$1.000 \times 10^{3}\ \mathrm{kg/m^3}$
Heat of fusion (0°C)	333 kJ/kg (80 kcal/kg)
Heat of vaporization (100°C)	2260 kJ/kg (539 kcal/kg)
Specific heat (15°C)	4186 J/kg·C° (1.00 kcal/kg·C°)
Index of refraction	1.33

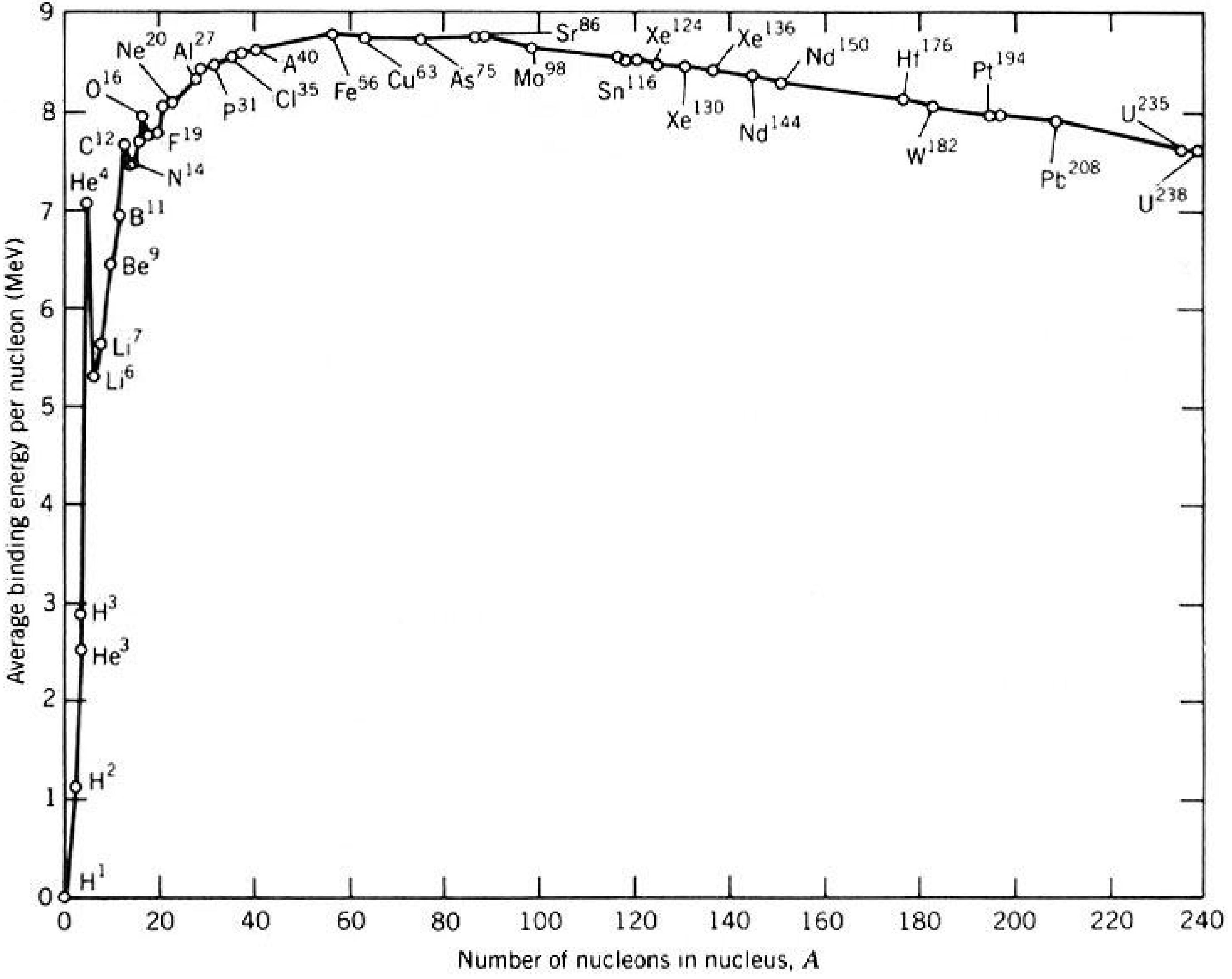
Periodic Table of the Elements[§]

Symbol — **Cl** 17 — Atomic Number
Atomic Mass[§] — 35.453
$3p^5$ — Electron Configuration (outer shells only)

Group I	Group II	Transition Elements										Group III	Group IV	Group V	Group VI	Group VII	Group VIII
H 1 1.00794 $1s^{1}$																	**He** 2 4.002602 $1s^{2}$
Li 3 6.941 $2s^{1}$	**Be** 4 9.012182 $2s^{2}$											**B** 5 10.811 $2p^{1}$	**C** 6 12.0107 $2p^{2}$	**N** 7 14.0067 $2p^{3}$	**O** 8 15.9994 $2p^{4}$	**F** 9 18.9984032 $2p^{5}$	**Ne** 10 20.1797 $2p^{6}$
Na 11 22.98976928 $3s^{1}$	**Mg** 12 24.3050 $3s^{2}$											**Al** 13 26.9815386 $3p^{1}$	**Si** 14 28.0855 $3p^{2}$	**P** 15 30.973762 $3p^{3}$	**S** 16 32.065 $3p^{4}$	**Cl** 17 35.453 $3p^{5}$	**Ar** 18 39.948 $3p^{6}$
K 19 39.0983 $4s^{1}$	**Ca** 20 40.078 $4s^{2}$	**Sc** 21 44.955912 $3d^{1}4s^{2}$	**Ti** 22 47.867 $3d^{2}4s^{2}$	**V** 23 50.9415 $3d^{3}4s^{2}$	**Cr** 24 51.9961 $3d^{5}4s^{1}$	**Mn** 25 54.938045 $3d^{5}4s^{2}$	**Fe** 26 55.845 $3d^{6}4s^{2}$	**Co** 27 58.933195 $3d^{7}4s^{2}$	**Ni** 28 58.6934 $3d^{8}4s^{2}$	**Cu** 29 63.546 $3d^{10}4s^{1}$	**Zn** 30 65.409 $3d^{10}4s^{2}$	**Ga** 31 69.723 $4p^{1}$	**Ge** 32 72.64 $4p^{2}$	**As** 33 74.92160 $4p^{3}$	**Se** 34 78.96 $4p^{4}$	**Br** 35 79.904 $4p^{5}$	**Kr** 36 83.798 $4p^{6}$
Rb 37 85.4678 $5s^{1}$	**Sr** 38 87.62 $5s^{2}$	**Y** 39 88.90585 $4d^{1}5s^{2}$	**Zr** 40 91.224 $4d^{2}5s^{2}$	**Nb** 41 92.90638 $4d^{4}5s^{1}$	**Mo** 42 95.94 $4d^{5}5s^{1}$	**Tc** 43 (98) $4d^{5}5s^{2}$	**Ru** 44 101.07 $4d^{7}5s^{1}$	**Rh** 45 102.90550 $4d^{8}5s^{1}$	**Pd** 46 106.42 $4d^{10}5s^{0}$	**Ag** 47 107.8682 $4d^{10}5s^{1}$	**Cd** 48 112.411 $4d^{10}5s^{2}$	**In** 49 114.818 $5p^{1}$	**Sn** 50 118.710 $5p^{2}$	**Sb** 51 121.760 $5p^{3}$	**Te** 52 127.60 $5p^{4}$	**I** 53 126.90447 $5p^{5}$	**Xe** 54 131.293 $5p^{6}$
Cs 55 132.9054519 $6s^{1}$	**Ba** 56 137.327 $6s^{2}$	57–71†	**Hf** 72 178.49 $5d^{2}6s^{2}$	**Ta** 73 180.94788 $5d^{3}6s^{2}$	**W** 74 183.84 $5d^{4}6s^{2}$	**Re** 75 186.207 $5d^{5}6s^{2}$	**Os** 76 190.23 $5d^{6}6s^{2}$	**Ir** 77 192.217 $5d^{7}6s^{2}$	**Pt** 78 195.084 $5d^{9}6s^{1}$	**Au** 79 196.966569 $5d^{10}6s^{1}$	**Hg** 80 200.59 $5d^{10}6s^{2}$	**Tl** 81 204.3833 $6p^{1}$	**Pb** 82 207.2 $6p^{2}$	**Bi** 83 208.98040 $6p^{3}$	**Po** 84 (209) $6p^{4}$	**At** 85 (210) $6p^{5}$	**Rn** 86 (222) $6p^{6}$
Fr 87 (223) $7s^{1}$	**Ra** 88 (226) $7s^{2}$	89–103‡	**Rf** 104 (267) $6d^{2}7s^{2}$	**Db** 105 (268) $6d^{3}7s^{2}$	**Sg** 106 (271) $6d^{4}7s^{2}$	**Bh** 107 (272) $6d^{5}7s^{2}$	**Hs** 108 (277) $6d^{6}7s^{2}$	**Mt** 109 (276) $6d^{7}7s^{2}$	**Ds** 110 (281) $6d^{9}7s^{1}$	**Rg** 111 (280) $6d^{10}7s^{1}$	112 (285) $6d^{10}7s^{2}$						

†**Lanthanide Series**	**La** 57 138.90547 $5d^{1}6s^{2}$	**Ce** 58 140.116 $4f^{1}5d^{1}6s^{2}$	**Pr** 59 140.90765 $4f^{3}5d^{0}6s^{2}$	**Nd** 60 144.242 $4f^{4}5d^{0}6s^{2}$	**Pm** 61 (145) $4f^{5}5d^{0}6s^{2}$	**Sm** 62 150.36 $4f^{6}5d^{0}6s^{2}$	**Eu** 63 151.964 $4f^{7}5d^{0}6s^{2}$	**Gd** 64 157.25 $4f^{7}5d^{1}6s^{2}$	**Tb** 65 158.92535 $4f^{9}5d^{0}6s^{2}$	**Dy** 66 162.500 $4f^{10}5d^{0}6s^{2}$	**Ho** 67 164.93032 $4f^{11}5d^{0}6s^{2}$	**Er** 68 167.259 $4f^{12}5d^{0}6s^{2}$	**Tm** 69 168.93421 $4f^{13}5d^{0}6s^{2}$	**Yb** 70 173.04 $4f^{14}5d^{0}6s^{2}$	**Lu** 71 174.967 $4f^{14}5d^{1}6s^{2}$
‡**Actinide Series**	**Ac** 89 (227) $6d^{1}7s^{2}$	**Th** 90 232.03806 $6d^{2}7s^{2}$	**Pa** 91 231.03588 $5f^{2}6d^{1}7s^{2}$	**U** 92 238.0289 $5f^{3}6d^{1}7s^{2}$	**Np** 93 (237) $5f^{4}6d^{1}7s^{2}$	**Pu** 94 (244) $5f^{6}6d^{0}7s^{2}$	**Am** 95 (243) $5f^{7}6d^{0}7s^{2}$	**Cm** 96 (247) $5f^{7}6d^{1}7s^{2}$	**Bk** 97 (247) $5f^{9}6d^{0}7s^{2}$	**Cf** 98 (251) $5f^{10}6d^{0}7s^{2}$	**Es** 99 (252) $5f^{11}6d^{0}7s^{2}$	**Fm** 100 (257) $5f^{12}6d^{0}7s^{2}$	**Md** 101 (258) $5f^{13}6d^{0}7s^{2}$	**No** 102 (259) $5f^{14}6d^{0}7s^{2}$	**Lr** 103 (262) $5f^{14}6d^{1}7s^{2}$

[§] Atomic mass values averaged over isotopes in percentages they occur on Earth's surface. For many unstable elements, mass of the longest-lived known isotope is given in parentheses. 2006 revisions. (See also Appendix F.) Preliminary evidence (unconfirmed) has been reported for elements 113, 114, 115, 116 and 118.


Average binding energy per nucleon (MeV)
Number of nucleons in nucleus, A
0
1
2
3
4
5
6
7
8
9
0
20
40
60
80
100
120
140
160
180
200
220
240
H^{1}
H^{2}
He^{3}
H^{3}
He^{4}
Li^{6}
Li^{7}
Be^{9}
B^{11}
C^{12}
N^{14}
O^{16}
F^{19}
Ne^{20}
Al^{27}
P^{31}
Cl^{35}
A^{40}
Fe^{56}
Cu^{63}
As^{75}
Sr^{86}
Mo^{98}
Sn^{116}
Xe^{124}
Xe^{130}
Xe^{136}
Nd^{144}
Nd^{150}
Hf^{176}
W^{182}
Pt^{194}
Pt^{208}
U^{235}
U^{238}